IDENTITA' VETTORIALI

Se $\mathbf{u} = u_x\,\hat{\mathbf{x}} + u_y\,\hat{\mathbf{y}} + u_z\,\hat{\mathbf{z}}$
$\mathbf{v} = v_x\,\hat{\mathbf{x}} + v_y\,\hat{\mathbf{y}} + v_z\,\hat{\mathbf{z}}$
$\mathbf{w} = w_x\,\hat{\mathbf{x}} + w_y\,\hat{\mathbf{y}} + w_z\,\hat{\mathbf{z}}$

allora

(prodotto scalare) $\mathbf{u}\cdot\mathbf{v} = u_x v_x + u_y v_y + u_z v_z$

(prodotto vettoriale) $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} = (u_y v_z - u_z v_y)\,\hat{\mathbf{x}} + (u_z v_x - u_x v_z)\,\hat{\mathbf{y}} + (u_x v_y - u_y v_x)\,\hat{\mathbf{z}}$

modulo di $\mathbf{u}$ $= |\mathbf{u}| = \sqrt{\mathbf{u}\cdot\mathbf{u}} = \sqrt{u_x^2 + u_y^2 + u_z^2}$

angolo compreso fra $\mathbf{u}$ e $\mathbf{v}$ $= \cos^{-1}\left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$

identità dei prodotti tripli: $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \mathbf{v}\cdot(\mathbf{w}\times\mathbf{u}) = \mathbf{w}\cdot(\mathbf{u}\times\mathbf{v})$ $\quad \mathbf{u}\times(\mathbf{v}\times\mathbf{w}) = (\mathbf{u}\cdot\mathbf{w})\mathbf{v} - (\mathbf{u}\cdot\mathbf{v})\mathbf{w}$

IDENTITA' CONTENENTI GRADIENTE, DIVERGENZA, ROTORE E LAPLACIANO

$\boldsymbol{\nabla} = \hat{\mathbf{x}}\dfrac{\partial}{\partial x} + \hat{\mathbf{y}}\dfrac{\partial}{\partial y} + \hat{\mathbf{z}}\dfrac{\partial}{\partial z}$ operatore “nabla” o “del”

$\mathbf{F}(x,y,z) = F_x(x,y,z)\,\hat{\mathbf{x}} + F_y(x,y,z)\,\hat{\mathbf{y}} + F_z(x,y,z)\,\hat{\mathbf{z}}$

$\boldsymbol{\nabla}\phi(x,y,z) = \mathbf{grad}\,\phi(x,y,z) = \dfrac{\partial\phi}{\partial x}\hat{\mathbf{x}} + \dfrac{\partial\phi}{\partial y}\hat{\mathbf{y}} + \dfrac{\partial\phi}{\partial z}\hat{\mathbf{z}}$

$\boldsymbol{\nabla}\cdot\mathbf{F}(x,y,z) = \textbf{div}\,\mathbf{F}(x,y,z) = \dfrac{\partial F_x}{\partial x} + \dfrac{\partial F_y}{\partial y} + \dfrac{\partial F_z}{\partial z}$

$$\boldsymbol{\nabla}\times\mathbf{F}(x,y,z) = \mathbf{rot}\,\mathbf{F}(x,y,z) = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{\mathbf{z}}$$

$\mathbf{a}\cdot\boldsymbol{\nabla} f = a_x\dfrac{\partial f}{\partial x} + a_y\dfrac{\partial f}{\partial y} + a_z\dfrac{\partial f}{\partial z}$

$(\mathbf{a}\cdot\boldsymbol{\nabla})\mathbf{F} = \big(\mathbf{a}\cdot\boldsymbol{\nabla}F_x\big)\,\hat{\mathbf{x}} + \big(\mathbf{a}\cdot\boldsymbol{\nabla}F_y\big)\,\hat{\mathbf{y}} + \big(\mathbf{a}\cdot\boldsymbol{\nabla}F_z\big)\,\hat{\mathbf{z}}$

$\boldsymbol{\nabla}(\phi\psi) = \phi\boldsymbol{\nabla}\psi + \psi\boldsymbol{\nabla}\phi$

$\boldsymbol{\nabla}\cdot(\mathbf{F}\times\mathbf{G}) = (\boldsymbol{\nabla}\times\mathbf{F})\cdot\mathbf{G} - \mathbf{F}\cdot(\boldsymbol{\nabla}\times\mathbf{G})$

$\boldsymbol{\nabla}\cdot(\phi\mathbf{F}) = (\boldsymbol{\nabla}\phi)\cdot\mathbf{F} + \phi\,(\boldsymbol{\nabla}\cdot\mathbf{F})$

$\boldsymbol{\nabla}\times(\mathbf{F}\times\mathbf{G}) = \mathbf{F}(\boldsymbol{\nabla}\cdot\mathbf{G}) - \mathbf{G}\,(\boldsymbol{\nabla}\cdot\mathbf{F}) - (\mathbf{F}\cdot\boldsymbol{\nabla})\mathbf{G} + (\mathbf{G}\cdot\boldsymbol{\nabla})\mathbf{F}$

$\boldsymbol{\nabla}\times(\phi\mathbf{F}) = (\boldsymbol{\nabla}\phi)\times\mathbf{F} + \phi\,(\boldsymbol{\nabla}\times\mathbf{F})$

$\boldsymbol{\nabla}(\mathbf{F}\cdot\mathbf{G}) = \mathbf{F}\times(\boldsymbol{\nabla}\times\mathbf{G}) + \mathbf{G}\times(\boldsymbol{\nabla}\times\mathbf{F}) + (\mathbf{F}\cdot\boldsymbol{\nabla})\mathbf{G} + (\mathbf{G}\cdot\boldsymbol{\nabla})\mathbf{F}$

$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\phi) = \mathbf{0}$ $\quad$ ($\mathbf{rot}\,\mathbf{grad} = \mathbf{0}$)

$\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\times\mathbf{F}) = 0$ $\quad$ ($\textbf{div}\,\mathbf{rot} = 0$)

$\nabla^2\phi(x,y,z) = \boldsymbol{\nabla}\cdot\boldsymbol{\nabla}\phi(x,y,z) = \textbf{div}\,\mathbf{grad}\,\phi = \dfrac{\partial^2\phi}{\partial x^2} + \dfrac{\partial^2\phi}{\partial y^2} + \dfrac{\partial^2\phi}{\partial z^2}$

$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{F}) = \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{F}) - \nabla^2\mathbf{F}$ $\quad$ ($\mathbf{rot}\,\mathbf{rot} = \mathbf{grad}\,\textbf{div}$ $-$ laplaciano)

VERSIONI DEL TEOREMA FONDAMENTALE DEL CALCOLO DIFFERENZIALE

$\displaystyle\int_a^b f'(t)\,dt = f(b) - f(a)$ $\quad$ (**teorema fondamentale** in una dimensione)

$\displaystyle\int_C \boldsymbol{\nabla}\phi\cdot d\mathbf{r} = \phi\big(\mathbf{r}(b)\big) - \phi\big(\mathbf{r}(a)\big)$ se C è la curva $\mathbf{r} = \mathbf{r}(t)$, $(a \le t \le b)$

$\displaystyle\iint_R \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) dA = \oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C \big(F_x(x,y)\,dx + F_y(x,y)\,dy\big)$ dove C è il contorno di R orientato positivamente $\quad$ **(teorema di Green)**

$\displaystyle\iint_S \boldsymbol{\nabla}\times\mathbf{F}\cdot\hat{\mathbf{n}}\,dS = \oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C \big(F_x(x,y,z)\,dx + F_y(x,y,z)\,dy + F_z(x,y,z)\,dz\big)$ dove C è il contorno orientato di S $\quad$ **(teorema di Stokes)**

Versioni tridimensionali: S è il contorno chiuso di V, con vettore normale esterno $\hat{\mathbf{n}}$

$\displaystyle\iiint_V \boldsymbol{\nabla}\cdot\mathbf{F}\,dV = \oiint_S \mathbf{F}\cdot\hat{\mathbf{n}}\,dS$ $\qquad$ $\displaystyle\int_V \boldsymbol{\nabla}\cdot\mathbf{F} = \oint_S \mathbf{F}\cdot\hat{\mathbf{n}}$ $\qquad$ **(teorema della divergenza)**

$\displaystyle\iiint_V \boldsymbol{\nabla}\phi\,dV = \oiint_S \phi\,\hat{\mathbf{n}}\,dS$ $\qquad$ $\displaystyle\int_V \boldsymbol{\nabla}\phi = \oint_S \phi\,\hat{\mathbf{n}}$ $\qquad$ **(teorema del gradiente)**

$\displaystyle\iiint_V \boldsymbol{\nabla}\times\mathbf{F}\,dV = -\oiint_S \mathbf{F}\times\hat{\mathbf{n}}\,dS$ $\qquad$ $\displaystyle\int_V \boldsymbol{\nabla}\times\mathbf{F} = -\oint_S \mathbf{F}\times\hat{\mathbf{n}}$ $\qquad$ **(teorema del rotore)**

COORDINATE POLARI PIANE

vettore posizione: $\mathbf{r} = r\cos\theta\,\hat{\mathbf{x}} + r\sin\theta\,\hat{\mathbf{y}}$

fattori di scala: $h_r = \left|\frac{\partial \mathbf{r}}{\partial r}\right| = 1,\quad h_\theta = \left|\frac{\partial \mathbf{r}}{\partial \theta}\right| = r$

trasformazione: $r = \sqrt{x^2+y^2}$ $\qquad x = r\cos\theta$

$\theta = \tan2^{-1}(y,x)$ $\qquad y = r\sin\theta$

base locale: $\hat{\mathbf{r}}(\theta) = \cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}}$ $\qquad \hat{\mathbf{x}} = \cos\theta\,\hat{\mathbf{r}}(\theta) - \sin\theta\,\hat{\boldsymbol{\theta}}(\theta)$

$\hat{\boldsymbol{\theta}}(\theta) = -\sin\theta\,\hat{\mathbf{x}} + \cos\theta\,\hat{\mathbf{y}}$ $\qquad \hat{\mathbf{y}} = \sin\theta\,\hat{\mathbf{r}}(\theta) + \cos\theta\,\hat{\boldsymbol{\theta}}(\theta)$

elemento di area: $dV = r\,dr\,d\theta$

campo scalare: $f(r,\theta)$

campo vettoriale: $\mathbf{F}(r,\theta) = F_r(r,\theta)\,\hat{\mathbf{r}}(\theta) + F_\theta(r,\theta)\,\hat{\boldsymbol{\theta}}(\theta)$

gradiente: $\boldsymbol{\nabla} f = \frac{\partial f}{\partial r}\,\hat{\mathbf{r}}(\theta) + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\hat{\boldsymbol{\theta}}(\theta)$

divergenza: $\boldsymbol{\nabla}\cdot\mathbf{F} = \frac{1}{r}\frac{\partial}{\partial r}\big(rF_r\big) + \frac{1}{r}\frac{\partial F_\theta}{\partial\theta}$

rotore: $\boldsymbol{\nabla}\times\mathbf{F} = \left[\frac{1}{r}\frac{\partial}{\partial r}\big(rF_\theta\big) - \frac{1}{r}\frac{\partial F_r}{\partial\theta}\right]\hat{\mathbf{z}}$

laplaciano: $\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial\theta^2}$

lapl. vett.:
$$\nabla^2\mathbf{F} = \left[\nabla^2 F_r - \frac{F_r}{r^2} - \frac{2}{r^2}\frac{\partial F_\theta}{\partial\theta}\right]\hat{\mathbf{r}}(\theta) + \left[\nabla^2 F_\theta - \frac{F_\theta}{r^2} + \frac{2}{r^2}\frac{\partial F_r}{\partial\theta}\right]\hat{\boldsymbol{\theta}}(\theta)$$

advezione: $\mathbf{a}\cdot\boldsymbol{\nabla} f = a_r\frac{\partial f}{\partial r} + \frac{a_\theta}{r}\frac{\partial f}{\partial\theta}$

advez.:
$$(\mathbf{a}\cdot\boldsymbol{\nabla})\mathbf{F} = \left[a_r\frac{\partial F_r}{\partial r} + \frac{a_\theta}{r}\left(\frac{\partial F_r}{\partial\theta} - F_\theta\right)\right]\hat{\mathbf{r}}(\theta) + \left[a_r\frac{\partial F_\theta}{\partial r} + \frac{a_\theta}{r}\left(\frac{\partial F_\theta}{\partial\theta} + F_r\right)\right]\hat{\boldsymbol{\theta}}(\theta)$$

COORDINATE CILINDRICHE

vettore posizione: $\mathbf{r} = R\cos\theta\,\hat{\mathbf{x}} + R\sin\theta\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$

fattori di scala: $h_R = \left|\frac{\partial \mathbf{r}}{\partial R}\right| = 1,\quad h_\theta = \left|\frac{\partial \mathbf{r}}{\partial \theta}\right| = R,\quad h_z = \left|\frac{\partial \mathbf{r}}{\partial z}\right| = 1$

trasformazione: $R = \sqrt{x^2+y^2}$ $\qquad x = R\cos\theta$

$\theta = \tan2^{-1}(y,x)$ $\qquad y = R\sin\theta$

$z = z$ $\qquad z = z$

base locale: $\hat{\mathbf{R}}(\theta) = \cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}}$ $\qquad \hat{\mathbf{x}} = \cos\theta\,\hat{\mathbf{R}}(\theta) - \sin\theta\,\hat{\boldsymbol{\theta}}(\theta)$

$\hat{\boldsymbol{\theta}}(\theta) = -\sin\theta\,\hat{\mathbf{x}} + \cos\theta\,\hat{\mathbf{y}}$ $\qquad \hat{\mathbf{y}} = \sin\theta\,\hat{\mathbf{R}}(\theta) + \cos\theta\,\hat{\boldsymbol{\theta}}(\theta)$

elemento di volume: $dV = R\,dR\,d\theta\,dz$

campo scalare: $f(R,\theta,z)$

campo vett.: $\mathbf{F}(R,\theta,z) = F_R(R,\theta,z)\,\hat{\mathbf{R}}(\theta) + F_\theta(R,\theta,z)\,\hat{\boldsymbol{\theta}}(\theta) + F_z(R,\theta,z)\,\hat{\mathbf{z}}$

gradiente: $\boldsymbol{\nabla} f = \frac{\partial f}{\partial R}\,\hat{\mathbf{R}}(\theta) + \frac{1}{R}\frac{\partial f}{\partial\theta}\,\hat{\boldsymbol{\theta}}(\theta) + \frac{\partial f}{\partial z}\,\hat{\mathbf{z}}$

divergenza: $\boldsymbol{\nabla}\cdot\mathbf{F} = \frac{1}{R}\frac{\partial}{\partial R}\big(RF_R\big) + \frac{1}{R}\frac{\partial F_\theta}{\partial\theta} + \frac{\partial F_z}{\partial z}$

rotore:
$$\boldsymbol{\nabla}\times\mathbf{F} = \left[\frac{1}{R}\frac{\partial F_z}{\partial\theta} - \frac{\partial F_\theta}{\partial z}\right]\hat{\mathbf{R}}(\theta) + \left[\frac{\partial F_R}{\partial z} - \frac{\partial F_z}{\partial R}\right]\hat{\boldsymbol{\theta}}(\theta) + \left[\frac{1}{R}\frac{\partial}{\partial R}\big(RF_\theta\big) - \frac{1}{R}\frac{\partial F_R}{\partial\theta}\right]\hat{\mathbf{z}}$$

laplaciano: $\nabla^2 f = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial f}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2 f}{\partial\theta^2} + \frac{\partial^2 f}{\partial z^2}$

lapl. vett.:
$$\nabla^2\mathbf{F} = \left[\nabla^2 F_R - \frac{F_R}{R^2} - \frac{2}{R^2}\frac{\partial F_\theta}{\partial\theta}\right]\hat{\mathbf{R}}(\theta) + \left[\nabla^2 F_\theta - \frac{F_\theta}{R^2} + \frac{2}{R^2}\frac{\partial F_R}{\partial\theta}\right]\hat{\boldsymbol{\theta}}(\theta) + \left[\nabla^2 F_z\right]\hat{\mathbf{z}}$$

advezione: $\mathbf{a}\cdot\boldsymbol{\nabla} f = a_R\frac{\partial f}{\partial R} + \frac{a_\theta}{R}\frac{\partial f}{\partial\theta} + a_z\frac{\partial f}{\partial z}$

advez.:
$$(\mathbf{a}\cdot\boldsymbol{\nabla})\mathbf{F} = \left[\mathbf{a}\cdot\boldsymbol{\nabla}F_R - \frac{a_\theta F_\theta}{R}\right]\hat{\mathbf{R}}(\theta) + \left[\mathbf{a}\cdot\boldsymbol{\nabla}F_\theta + \frac{a_\theta F_R}{R}\right]\hat{\boldsymbol{\theta}}(\theta) + \left[\mathbf{a}\cdot\boldsymbol{\nabla}F_z\right]\hat{\mathbf{z}}$$
$$= \left[a_R\frac{\partial F_R}{\partial R} + \frac{a_\theta}{R}\left(\frac{\partial F_R}{\partial\theta} - F_\theta\right) + a_z\frac{\partial F_R}{\partial z}\right]\hat{\mathbf{R}}(\theta) + \left[a_R\frac{\partial F_\theta}{\partial R} + \frac{a_\theta}{R}\left(\frac{\partial F_\theta}{\partial\theta} + F_R\right) + a_z\frac{\partial F_\theta}{\partial z}\right]\hat{\boldsymbol{\theta}}(\theta) + \left[a_R\frac{\partial F_z}{\partial R} + \frac{a_\theta}{R}\frac{\partial F_z}{\partial\theta} + a_z\frac{\partial F_z}{\partial z}\right]\hat{\mathbf{z}}$$

COORDINATE SFERICHE

vettore posizione: $\mathbf{r} = r\sin\theta\cos\phi\,\hat{\mathbf{x}} + r\sin\theta\sin\phi\,\hat{\mathbf{y}} + r\cos\theta\,\hat{\mathbf{z}}$

fattori di scala: $h_r = \left|\frac{\partial\mathbf{r}}{\partial r}\right| = 1,\ h_\theta = \left|\frac{\partial\mathbf{r}}{\partial\theta}\right| = r,\ h_\phi = \left|\frac{\partial\mathbf{r}}{\partial\phi}\right| = r\sin\theta$

trasformazione:
$$r = \sqrt{x^2+y^2+z^2}$$
$$\theta = \cos^{-1}\left(z/\sqrt{x^2+y^2+z^2}\right)$$
$$\phi = \tan2^{-1}(y,x)$$
$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

base locale:
$$\hat{\mathbf{r}}(\theta,\phi) = \sin\theta\,\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}}$$
$$\hat{\boldsymbol{\theta}}(\theta,\phi) = \cos\theta\,\cos\phi\,\hat{\mathbf{x}} + \cos\theta\,\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}}$$
$$\hat{\boldsymbol{\phi}}(\phi) = -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}}$$

$$\hat{\mathbf{x}} = \sin\theta\cos\phi\,\hat{\mathbf{r}}(\theta,\phi) + \cos\theta\cos\phi\,\hat{\boldsymbol{\theta}}(\theta,\phi) - \sin\phi\,\hat{\boldsymbol{\phi}}(\phi)$$
$$\hat{\mathbf{y}} = \sin\theta\sin\phi\,\hat{\mathbf{r}}(\theta,\phi) + \cos\theta\sin\phi\,\hat{\boldsymbol{\theta}}(\theta,\phi) + \cos\phi\,\hat{\boldsymbol{\phi}}(\phi)$$
$$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}}(\theta,\phi) - \sin\theta\,\hat{\boldsymbol{\theta}}(\theta,\phi)$$

elemento di volume: $dV = r^2\sin\theta\,dr\,d\theta\,d\phi$

campo scalare: $f(r,\theta,\phi)$

c. vett.: $\mathbf{F}(r,\theta,\phi) = F_r(r,\theta,\phi)\,\hat{\mathbf{r}}(\theta,\phi) + F_\theta(r,\theta,\phi)\,\hat{\boldsymbol{\theta}}(\theta,\phi) + F_\phi(r,\theta,\phi)\,\hat{\boldsymbol{\phi}}(\phi)$

gradiente: $\nabla f = \frac{\partial f}{\partial r}\,\hat{\mathbf{r}}(\theta,\phi) + \frac{1}{r}\frac{\partial f}{\partial\theta}\,\hat{\boldsymbol{\theta}}(\theta,\phi) + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\,\hat{\boldsymbol{\phi}}(\phi)$

divergenza: $\nabla\cdot\mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,F_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi}$

rotore:
$$\nabla\times\mathbf{F} = \left[\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,F_\phi\right) - \frac{1}{r\sin\theta}\frac{\partial F_\theta}{\partial\phi}\right]\hat{\mathbf{r}}(\theta,\phi) + \left[\frac{1}{r\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}\left(rF_\phi\right)\right]\hat{\boldsymbol{\theta}}(\theta,\phi) + \left[\frac{1}{r}\frac{\partial}{\partial r}\left(rF_\theta\right) - \frac{1}{r}\frac{\partial F_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}(\phi)$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

$$\nabla^2\mathbf{F} = \left[\nabla^2F_r - \frac{2F_r}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial(\sin\theta\,F_\theta)}{\partial\theta} - \frac{2}{r^2\sin\theta}\frac{\partial F_\phi}{\partial\phi}\right]\hat{\mathbf{r}}(\theta,\phi) + \left[\nabla^2F_\theta - \frac{F_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial F_\phi}{\partial\phi} + \frac{2}{r^2}\frac{\partial F_r}{\partial\theta}\right]\hat{\boldsymbol{\theta}}(\theta,\phi) + \left[\nabla^2F_\phi - \frac{F_\phi}{r^2\sin^2\theta} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial F_\theta}{\partial\phi} + \frac{2}{r^2\sin\theta}\frac{\partial F_r}{\partial\phi}\right]\hat{\boldsymbol{\phi}}(\phi)$$

advezione: $\mathbf{a}\cdot\nabla f = a_r\frac{\partial f}{\partial r} + \frac{a_\theta}{r}\frac{\partial f}{\partial\theta} + \frac{a_\phi}{r\sin\theta}\frac{\partial f}{\partial\phi}$

$$\begin{aligned}(\mathbf{a}\cdot\nabla)\mathbf{F} &= \left[\mathbf{a}\cdot\nabla F_r - \frac{a_\theta F_\theta + a_\phi F_\phi}{r}\right]\hat{\mathbf{r}}(\theta,\phi) + \left[\mathbf{a}\cdot\nabla F_\theta + \frac{a_\theta F_r - \cot\theta\,a_\phi F_\phi}{r}\right]\hat{\boldsymbol{\theta}}(\theta,\phi) + \left[\mathbf{a}\cdot\nabla F_\phi + \frac{a_\phi(F_r + \cot\theta\,F_\theta)}{r}\right]\hat{\boldsymbol{\phi}}(\phi)\\
&= \left[a_r\frac{\partial F_r}{\partial r} + \frac{a_\theta}{r}\left(\frac{\partial F_r}{\partial\theta} - F_\theta\right) + \frac{a_\phi}{r}\left(\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - F_\phi\right)\right]\hat{\mathbf{r}}(\theta,\phi)\\
&+ \left[a_r\frac{\partial F_\theta}{\partial r} + \frac{a_\theta}{r}\left(\frac{\partial F_\theta}{\partial\theta} + F_r\right) + \frac{a_\phi}{r\sin\theta}\left(\frac{\partial F_\theta}{\partial\phi} - \cos\theta\,F_\phi\right)\right]\hat{\boldsymbol{\theta}}(\theta,\phi)\\
&+ \left[a_r\frac{\partial F_\phi}{\partial r} + \frac{a_\theta}{r}\frac{\partial F_\phi}{\partial\theta} + \frac{a_\phi}{r}\left[\frac{1}{\sin\theta}\left(\frac{\partial F_\phi}{\partial\phi} + \cos\theta\,F_\theta\right) + F_r\right]\right]\hat{\boldsymbol{\phi}}(\phi)\end{aligned}$$

COORDINATE CILINDRICHE PER PROBLEMI ASSISIMMETRICI CON EVENTUALE “SWIRL”

versori: $\hat{\mathbf{R}}(\theta) \to \hat{\mathbf{R}}$ e $\hat{\boldsymbol{\theta}}(\theta) \to \hat{\boldsymbol{\theta}}$

campo scalare: $f(R,z)$

gradiente: $\nabla f = \dfrac{\partial f}{\partial R}\,\hat{\mathbf{R}} + \dfrac{\partial f}{\partial z}\,\hat{\mathbf{z}}$

laplaciano: $\nabla^2 f = \dfrac{1}{R}\dfrac{\partial}{\partial R}\left(R\,\dfrac{\partial f}{\partial R}\right) + \dfrac{\partial^2 f}{\partial z^2}$

advezione: $\mathbf{a}\cdot\nabla f = a_R\dfrac{\partial f}{\partial R} + a_z\dfrac{\partial f}{\partial z}$

campo vettoriale: $\mathbf{F}(R,z) = F_R(R,z)\,\hat{\mathbf{R}} + F_z(R,z)\,\hat{\mathbf{z}} + F_\theta(R,z)\,\hat{\boldsymbol{\theta}}$

divergenza: $\nabla\cdot\mathbf{F} = \dfrac{1}{R}\dfrac{\partial}{\partial R}\big(RF_R\big) + \dfrac{\partial F_z}{\partial z}$

rotore: $\nabla\times\mathbf{F} = -\dfrac{\partial F_\theta}{\partial z}\,\hat{\mathbf{R}} + \dfrac{1}{R}\dfrac{\partial}{\partial R}\big(RF_\theta\big)\,\hat{\mathbf{z}} + \left[\dfrac{\partial F_R}{\partial z} - \dfrac{\partial F_z}{\partial R}\right]\hat{\boldsymbol{\theta}}$

lapl. vett.: $\nabla^2\mathbf{F} = \left[\nabla^2 F_R - \dfrac{F_R}{R^2}\right]\hat{\mathbf{R}} + \big[\nabla^2 F_z\big]\,\hat{\mathbf{z}} + \left[\nabla^2 F_\theta - \dfrac{F_\theta}{R^2}\right]\hat{\boldsymbol{\theta}}$

advez.:
$$
\begin{aligned}
(\mathbf{a}\cdot\nabla)\mathbf{F} &= \left[\mathbf{a}\cdot\nabla F_R - \frac{a_\theta F_\theta}{R}\right]\hat{\mathbf{R}} + \big[\mathbf{a}\cdot\nabla F_z\big]\,\hat{\mathbf{z}} + \left[\mathbf{a}\cdot\nabla F_\theta + \frac{a_\theta F_R}{R}\right]\hat{\boldsymbol{\theta}} \\
&= \left[a_R\frac{\partial F_R}{\partial R} + a_z\frac{\partial F_R}{\partial z} - \frac{a_\theta F_\theta}{R}\right]\hat{\mathbf{R}} + \left[a_R\frac{\partial F_z}{\partial R} + a_z\frac{\partial F_z}{\partial z}\right]\hat{\mathbf{z}} \\
&\quad + \left[a_R\frac{\partial F_\theta}{\partial R} + a_z\frac{\partial F_\theta}{\partial z} + \frac{a_\theta F_R}{R}\right]\hat{\boldsymbol{\theta}}
\end{aligned}
$$

COORDINATE SFERICHE PER PROBLEMI ASSISIMMETRICI CON EVENTUALE “SWIRL”

versori: $\hat{\mathbf{r}}(\theta,\phi) \to \hat{\mathbf{r}}(\theta)$, $\hat{\boldsymbol{\theta}}(\theta,\phi) \to \hat{\boldsymbol{\theta}}(\theta)$ e $\hat{\boldsymbol{\phi}}(\phi) \to \hat{\boldsymbol{\phi}}$

campo scalare: $f(r,\theta)$

gradiente: $\nabla f = \dfrac{\partial f}{\partial r}\,\hat{\mathbf{r}}(\theta) + \dfrac{1}{r}\dfrac{\partial f}{\partial \theta}\,\hat{\boldsymbol{\theta}}(\theta)$

laplaciano: $\nabla^2 f = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\,\dfrac{\partial f}{\partial r}\right) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta\,\dfrac{\partial f}{\partial\theta}\right)$

advezione: $\mathbf{a}\cdot\nabla f = a_r\dfrac{\partial f}{\partial r} + \dfrac{a_\theta}{r}\dfrac{\partial f}{\partial\theta}$

campo vett.: $\mathbf{F}(r,\theta) = F_r(r,\theta)\,\hat{\mathbf{r}}(\theta) + F_\theta(r,\theta)\,\hat{\boldsymbol{\theta}}(\theta) + F_\phi(r,\theta)\,\hat{\boldsymbol{\phi}}$

divergenza: $\nabla\cdot\mathbf{F} = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\big(r^2 F_r\big) + \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial\theta}\big(\sin\theta\,F_\theta\big)$

rotore:
$$
\begin{aligned}
\nabla\times\mathbf{F} &= \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\big(\sin\theta\,F_\phi\big)\,\hat{\mathbf{r}}(\theta) - \frac{1}{r}\frac{\partial}{\partial r}\big(rF_\phi\big)\,\hat{\boldsymbol{\theta}}(\theta) \\
&\quad + \left[\frac{1}{r}\frac{\partial}{\partial r}\big(rF_\theta\big) - \frac{1}{r}\frac{\partial F_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}
\end{aligned}
$$

lapl. vett.:
$$
\begin{aligned}
\nabla^2\mathbf{F} &= \left[\nabla^2 F_r - \frac{2F_r}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial(\sin\theta\,F_\theta)}{\partial\theta}\right]\hat{\mathbf{r}}(\theta) \\
&\quad + \left[\nabla^2 F_\theta - \frac{F_\theta}{r^2\sin^2\theta} + \frac{2}{r^2}\frac{\partial F_r}{\partial\theta}\right]\hat{\boldsymbol{\theta}}(\theta) \\
&\quad + \left[\nabla^2 F_\phi - \frac{F_\phi}{r^2\sin^2\theta}\right]\hat{\boldsymbol{\phi}}
\end{aligned}
$$

advez.:
$$
\begin{aligned}
(\mathbf{a}\cdot\nabla)\mathbf{F} &= \left[\mathbf{a}\cdot\nabla F_r - \frac{a_\theta F_\theta + a_\phi F_\phi}{r}\right]\hat{\mathbf{r}}(\theta) \\
&\quad + \left[\mathbf{a}\cdot\nabla F_\theta + \frac{a_\theta F_r - \cot\theta\,a_\phi F_\phi}{r}\right]\hat{\boldsymbol{\theta}}(\theta) \\
&\quad + \left[\mathbf{a}\cdot\nabla F_\phi + \frac{a_\phi(F_r + \cot\theta\,F_\theta)}{r}\right]\hat{\boldsymbol{\phi}} \\
&= \left[a_r\frac{\partial F_r}{\partial r} + \frac{a_\theta}{r}\left(\frac{\partial F_r}{\partial\theta} - F_\theta\right) - \frac{a_\phi F_\phi}{r}\right]\hat{\mathbf{r}}(\theta) \\
&\quad + \left[a_r\frac{\partial F_\theta}{\partial r} + \frac{a_\theta}{r}\left(\frac{\partial F_\theta}{\partial\theta} + F_r\right) - \cot\theta\,\frac{a_\phi F_\phi}{r}\right]\hat{\boldsymbol{\theta}}(\theta) \\
&\quad + \left[a_r\frac{\partial F_\phi}{\partial r} + \frac{a_\theta}{r}\frac{\partial F_\phi}{\partial\theta} + \frac{a_\phi}{r}(F_r + \cot\theta\,F_\theta)\right]\hat{\boldsymbol{\phi}}
\end{aligned}
$$

COORDINATE CURVILINEE ORTOGONALI

trasformazione: $x = x(u, v, w), \quad y = y(u, v, w), \quad z = z(u, v, w)$

vettore posizione: $\mathbf{r} = x(u, v, w)\,\hat{\mathbf{x}} + y(u, v, w)\,\hat{\mathbf{y}} + z(u, v, w)\,\hat{\mathbf{z}}$

fattori di scala: $h_u = \left|\frac{\partial \mathbf{r}}{\partial u}\right|, \quad h_v = \left|\frac{\partial \mathbf{r}}{\partial v}\right|, \quad h_w = \left|\frac{\partial \mathbf{r}}{\partial w}\right|$

base locale: $\hat{\mathbf{u}}(u, v, w) = \frac{1}{h_u}\frac{\partial \mathbf{r}}{\partial u}, \quad \hat{\mathbf{v}}(u, v, w) = \frac{1}{h_v}\frac{\partial \mathbf{r}}{\partial v}, \quad \hat{\mathbf{w}}(u, v, w) = \frac{1}{h_w}\frac{\partial \mathbf{r}}{\partial w}$

elemento di volume: $dV = h_u h_v h_w \, du \, dv \, dw$

campo scalare: $f(u, v, w)$

campo vettoriale: $\mathbf{F}(u, v, w) = F_u(u, v, w)\,\hat{\mathbf{u}} + F_v(u, v, w)\,\hat{\mathbf{v}} + F_w(u, v, w)\,\hat{\mathbf{w}}$

gradiente: $\boldsymbol{\nabla} f = \frac{1}{h_u}\frac{\partial f}{\partial u}\,\hat{\mathbf{u}} + \frac{1}{h_v}\frac{\partial f}{\partial v}\,\hat{\mathbf{v}} + \frac{1}{h_w}\frac{\partial f}{\partial w}\,\hat{\mathbf{w}}$

divergenza: $\boldsymbol{\nabla}\cdot\mathbf{F} = \frac{1}{h_u h_v h_w}\left[\frac{\partial}{\partial u}\big(h_v h_w F_u\big) + \frac{\partial}{\partial v}\big(h_u h_w F_v\big) + \frac{\partial}{\partial w}\big(h_u h_v F_w\big)\right]$

$$\nabla^2 f = \frac{1}{h_u h_v h_w}\left[\frac{\partial}{\partial u}\left(\frac{h_v h_w}{h_u}\frac{\partial f}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_u h_w}{h_v}\frac{\partial f}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_u h_v}{h_w}\frac{\partial f}{\partial w}\right)\right]$$

rotore: $\boldsymbol{\nabla}\times\mathbf{F} = \frac{1}{h_u h_v h_w}\begin{vmatrix} h_u\hat{\mathbf{u}} & h_v\hat{\mathbf{v}} & h_w\hat{\mathbf{w}} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_u F_u & h_v F_v & h_w F_w \end{vmatrix}$

EQUAZIONI DI LAPLACE E DI BERNOULLI PER CORRENTI INCOMPRIMIBILI IRROTAZIONALI DEI FLUIDI NON VISCOSI

$$\nabla^2\phi = 0$$

$$\frac{\partial \phi}{\partial n}_{|S} = b_n(\mathbf{r}_S, t)$$

$$\frac{\mathcal{P}(\mathbf{r},t)}{\overline{\rho}} = -\frac{\partial \phi(\mathbf{r},t)}{\partial t} - \frac{|\nabla\phi(\mathbf{r},t)|^2}{2} - \chi(\mathbf{r}) + C(t)$$

Condizione di compatibilità:

$$\oiint_S b_n(\mathbf{r}_S, t)\, dS = 0$$

EQUAZIONI DI EULERO PER CORRENTI INCOMPRIMIBILI DEI FLUIDI NON VISCOSI

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \frac{\nabla\mathcal{P}}{\overline{\rho}} = \mathbf{g}(\mathbf{r},t)$$

$$\nabla\cdot\mathbf{u} = 0$$

$$\mathbf{u}(\mathbf{r},0) = \mathbf{u}_0(\mathbf{r})$$

$$\hat{\mathbf{n}}\cdot\mathbf{u}(\mathbf{r},t)_{|S} = b_n(\mathbf{r}_S, t)$$

Condizioni di compatibilità:

$$\nabla\cdot\mathbf{u}_0 = 0$$

$$\oiint_S b_n(\mathbf{r}_S, t)\, dS = 0$$

$$\hat{\mathbf{n}}\cdot\mathbf{u}_0(\mathbf{r})_{|S} = b_n(\mathbf{r}_S, 0)$$

EQUAZIONI DI NAVIER–STOKES PER CORRENTI INCOMPRIMIBILI DEI FLUIDI VISCOSI (NEWTONIANI)

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} - \nu\nabla^2\mathbf{u} + \frac{\nabla\mathcal{P}}{\overline{\rho}} = \mathbf{g}(\mathbf{r},t) \qquad [\nu = \overline{\mu}/\overline{\rho}]$$

$$\nabla\cdot\mathbf{u} = 0$$

$$\mathbf{u}(\mathbf{r},0) = \mathbf{u}_0(\mathbf{r})$$

$$\mathbf{u}(\mathbf{r},t)_{|S} = \mathbf{b}(\mathbf{r}_S, t)$$

Condizioni di compatibilità:

$$\nabla\cdot\mathbf{u}_0 = 0$$

$$\oiint_S \hat{\mathbf{n}}\cdot\mathbf{b}(\mathbf{r}_S, t)\, dS = 0$$

$$\hat{\mathbf{n}}\cdot\mathbf{u}_0(\mathbf{r})_{|S} = \hat{\mathbf{n}}\cdot\mathbf{b}(\mathbf{r}_S, 0)$$

EQUAZIONI DI EULERO PER FLUIDI COMPRIMIBILI NON VISCOSI

Forma convettiva:

$$\frac{\partial \rho}{\partial t} + \mathbf{u}\cdot\nabla\rho + \rho\nabla\cdot\mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \frac{\nabla P}{\rho} = \mathbf{g}(\mathbf{r},t)$$

$$\frac{\partial e}{\partial t} + \mathbf{u}\cdot\nabla e + \frac{P}{\rho}\nabla\cdot\mathbf{u} = 0$$

$$P = P(e,\rho), \qquad T = T(e,\rho)$$

Forma intermedia:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0$$

$$\frac{\partial (\rho\mathbf{u})}{\partial t} + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u}) + \nabla P = \rho\mathbf{g}(\mathbf{r},t)$$

$$\frac{\partial (\rho e)}{\partial t} + \nabla\cdot(\rho e\mathbf{u}) + P\,\nabla\cdot\mathbf{u} = 0$$

$$P = P(e,\rho), \qquad T = T(e,\rho)$$

Forma conservativa con variabili conservative:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0$$

$$\frac{\partial (\rho\mathbf{u})}{\partial t} + \nabla\cdot\big(\rho\mathbf{u}\otimes\mathbf{u} + P\,\mathbb{I}\big) = \rho\mathbf{g}(\mathbf{r},t)$$

$$\frac{\partial (\rho e^{\mathrm{t}})}{\partial t} + \nabla\cdot\big((\rho e^{\mathrm{t}} + P)\mathbf{u}\big) = \rho\mathbf{u}\cdot\mathbf{g}(\mathbf{r},t)$$

$$P = P(e,\rho), \qquad T = T(e,\rho), \qquad e^{\mathrm{t}} = e^{\mathrm{tot}} = e + \tfrac{1}{2}|\mathbf{u}|^2$$

EQUAZIONI DI NAVIER–STOKES PER FLUIDI COMPRIMIBILI VISCOSI (NEWTONIANI)

Forma con energia interna:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0$$

$$\frac{\partial (\rho\mathbf{u})}{\partial t} + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u}) + \nabla P = \nabla\cdot\mathbb{S}(\mathbf{u}) + \rho\mathbf{g}(\mathbf{r},t)$$

$$\frac{\partial (\rho e)}{\partial t} + \nabla\cdot(\rho e\mathbf{u}) + P\,\nabla\cdot\mathbf{u} = \nabla\cdot(\kappa\nabla T) + \mathbb{S}(\mathbf{u}):\mathbb{E}(\mathbf{u})$$

$$\mathbb{S}(\mathbf{u}) = 2\mu\,\mathbb{E}(\mathbf{u}) + \lambda\,(\nabla\cdot\mathbf{u})\,\mathbb{I}$$

$$\mathbb{E}(\mathbf{u}) \longleftrightarrow e_{i,j}(\mathbf{u}) = \tfrac{1}{2}\big[\hat{\boldsymbol{\eta}}_i\cdot(\hat{\boldsymbol{\eta}}_j\cdot\nabla)\mathbf{u} + \hat{\boldsymbol{\eta}}_j\cdot(\hat{\boldsymbol{\eta}}_i\cdot\nabla)\mathbf{u}\big], \qquad i,j = 1,2,3$$

$$P = P(e,\rho), \qquad T = T(e,\rho)$$

Forma conservativa con variabili conservative:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0$$

$$\frac{\partial (\rho\mathbf{u})}{\partial t} + \nabla\cdot\big(\rho\mathbf{u}\otimes\mathbf{u} + P\,\mathbb{I}\big) = \nabla\cdot\mathbb{S}(\mathbf{u}) + \rho\mathbf{g}(\mathbf{r},t)$$

$$\frac{\partial (\rho e^{\mathrm{t}})}{\partial t} + \nabla\cdot\big((\rho e^{\mathrm{t}} + P)\mathbf{u}\big) = \nabla\cdot\big(\kappa\nabla T + \mathbf{u}\cdot\mathbb{S}(\mathbf{u})\big) + \rho\mathbf{u}\cdot\mathbf{g}(\mathbf{r},t)$$

$$\mathbb{S}(\mathbf{u}) = 2\mu\,\mathbb{E}(\mathbf{u}) + \lambda\,(\nabla\cdot\mathbf{u})\,\mathbb{I}$$

$$\mathbb{E}(\mathbf{u}) \longleftrightarrow e_{i,j}(\mathbf{u}) = \tfrac{1}{2}\big[\hat{\boldsymbol{\eta}}_i\cdot(\hat{\boldsymbol{\eta}}_j\cdot\nabla)\mathbf{u} + \hat{\boldsymbol{\eta}}_j\cdot(\hat{\boldsymbol{\eta}}_i\cdot\nabla)\mathbf{u}\big], \qquad i,j = 1,2,3$$

$$P = P(e,\rho), \qquad T = T(e,\rho), \qquad e^{\mathrm{t}} = e^{\mathrm{tot}} = e + \tfrac{1}{2}|\mathbf{u}|^2$$

EQUAZIONI DI EULERO DELLA GASDINAMICA IN FORMA CONSERVATIVA

Equazioni in una dimensione [$q = \rho u$]:

$$\rho_t + q_x = 0$$

$$q_t + \left(\frac{q^2}{\rho} + P\right)_x = 0$$

$$E^{\mathrm{t}}_t + \left(\left(E^{\mathrm{t}} + P\right)\frac{q}{\rho}\right)_x = 0$$

$$P = P\left(\frac{E^{\mathrm{t}}}{\rho} - \frac{|q|^2}{2\rho^2}, \rho\right) \equiv \Pi\left(\rho, q, E^{\mathrm{t}}\right)$$

Equazioni in una dimensione [$q = \rho u$]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q^2}{\rho} + P\right) = 0$$

$$\frac{\partial E^{\mathrm{t}}}{\partial t} + \frac{\partial}{\partial x}\left(\left(E^{\mathrm{t}} + P\right)\frac{q}{\rho}\right) = 0$$

$$P = P\left(\frac{E^{\mathrm{t}}}{\rho} - \frac{|q|^2}{2\rho^2}, \rho\right) \equiv \Pi\left(\rho, q, E^{\mathrm{t}}\right)$$

Equazioni in più dimensioni [$\mathbf{q} = \rho \mathbf{u}$]:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot \mathbf{q} = 0$$

$$\frac{\partial \mathbf{q}}{\partial t} + \boldsymbol{\nabla}\cdot\left(\frac{\mathbf{q}\otimes\mathbf{q}}{\rho} + P\,\mathbb{I}\right) = 0$$

$$\frac{\partial E^{\mathrm{t}}}{\partial t} + \boldsymbol{\nabla}\cdot\left(\left(E^{\mathrm{t}} + P\right)\frac{\mathbf{q}}{\rho}\right) = 0$$

$$P = P\left(\frac{E^{\mathrm{t}}}{\rho} - \frac{|\mathbf{q}|^2}{2\rho^2}, \rho\right) \equiv \Pi\left(\rho, \mathbf{q}, E^{\mathrm{t}}\right)$$

VETTORE DELLE INCOGNITE E FLUSSI DELLE EQUAZIONI DELLA GASDINAMICA

Problemi in una dimensione [$q = \rho u$]:

$$w = \begin{pmatrix} \rho \\ q \\ E^{\mathrm{t}} \end{pmatrix}, \qquad f(w) = \begin{pmatrix} q \\ \dfrac{q^2}{\rho} + P \\ \left(E^{\mathrm{t}} + P\right)\dfrac{q}{\rho} \end{pmatrix}$$

Problemi in più dimensioni [$\mathbf{q} = \rho \mathbf{u}$]:

$$w = \begin{pmatrix} \rho \\ \mathbf{q} \\ E^{\mathrm{t}} \end{pmatrix}, \qquad \mathbf{f}(w) = \begin{pmatrix} \mathbf{q} \\ \dfrac{\mathbf{q}\otimes\mathbf{q}}{\rho} + P\,\mathbb{I} \\ \left(E^{\mathrm{t}} + P\right)\dfrac{\mathbf{q}}{\rho} \end{pmatrix}$$

FORMA CONSERVATIVA E FORMA QUASI-LINEARE DELLE EQUAZIONI DELLA GASDINAMICA

Sistema iperbolico in una dimensione:

$$w_t + [f(w)]_x = 0$$

$$w_t + A(w)\, w_x = 0$$

$$A(w) = \frac{\partial f(w)}{\partial w}$$

Sistema iperbolico in una dimensione:

$$\frac{\partial w}{\partial t} + \frac{\partial f(w)}{\partial x} = 0$$

$$\frac{\partial w}{\partial t} + A(w)\,\frac{\partial w}{\partial x} = 0$$

$$A(w) = \frac{\partial f(w)}{\partial w}$$

Sistema iperbolico in più dimensioni:

$$\frac{\partial w}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{f}(w) = 0$$

$$\frac{\partial w}{\partial t} + \mathbf{A}(w)\cdot\boldsymbol{\nabla} w = 0$$

$$\mathbf{A}(w) = \frac{\partial \mathbf{f}(w)}{\partial w}$$

EQUAZIONI DELLA GASDINAMICA PER CORRENTI ISENTROPICHE IN FORMA CONSERVATIVA

Equazioni in una dimensione [$q = \rho u$]:

$$\rho_t + q_x = 0$$

$$q_t + \left(\frac{q^2}{\rho} + P(\rho)\right)_x = 0$$

$$[P = P(\rho) = P(\overline{s}, \rho)]$$

Equazioni in una dimensione [$q = \rho u$]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q^2}{\rho} + P(\rho)\right) = 0$$

$$[P = P(\rho) = P(\overline{s}, \rho)]$$

Equazioni in più dimensioni [$\mathbf{q} = \rho \mathbf{u}$]:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot \mathbf{q} = 0$$

$$\frac{\partial \mathbf{q}}{\partial t} + \boldsymbol{\nabla}\cdot\left(\frac{\mathbf{q}\otimes\mathbf{q}}{\rho} + P(\rho)\,\mathbb{I}\right) = 0$$

$$[P = P(\rho) = P(\overline{s}, \rho)]$$

EQUAZIONI DELLA GASDINAMICA PER GAS IDEALE ISOTERMO IN FORMA CONSERVATIVA

Equazioni in una dimensione [$q = \rho u$]:

$$\rho_t + q_x = 0$$

$$q_t + \left(\frac{q^2}{\rho} + \overline{a}^2\rho\right)_x = 0$$

$$\left[\overline{a}^2\rho = P(\rho)\right]$$

Equazioni in una dimensione [$q = \rho u$]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q^2}{\rho} + \overline{a}^2\rho\right) = 0$$

$$\left[\overline{a}^2\rho = P(\rho)\right]$$

Equazioni in più dimensioni [$\mathbf{q} = \rho \mathbf{u}$]:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot \mathbf{q} = 0$$

$$\frac{\partial \mathbf{q}}{\partial t} + \boldsymbol{\nabla}\cdot\left(\frac{\mathbf{q}\otimes\mathbf{q}}{\rho} + \overline{a}^2\rho\,\mathbb{I}\right) = 0$$

$$\left[\overline{a}^2\rho = P(\rho)\right]$$